

Magnetically Induced Plasma Rotation and Nuclear Fusion

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Introductory views are presented on the physically different but remarkably good fusion reaction performance of the dense plasma focus compared to the mainline magnetic fusion research assemblies. Reference is given to our attempt in explaining a reputed fundamental but not understood phenomenon in the focus plasma by an ion acceleration mechanism of basically the betatron type. Paradoxically, this magnetically induced plasma rotation is both contained in and denied by standard plasma equations, briefly derived and discussed with respect to validity. The neglect of the Hall term in the usual “Ideal MHD Equations” is shown to be in error with regard to current magnetic fusion research plasmas. Magnetically induced plasma rotation is derived from the ion fluid motion equation, from basic MHD fluid equations and by an analysis of the electro-magnetic torque distribution. General validity is proved by invoking usually overlooked consequences of the noncentral character of the forces sustaining quasi-neutrality. Arguments are given in favor of new approaches to magnetic fusion.

I. Introduction

There is general agreement about the capability of the dense plasma focus as an intense neutron source by fusion reactions [1], however, its extension so as to generate large-scale fusion energy remains in doubt. A focus device of modest laboratory size can for each discharge emit neutron bursts of $\approx 10^{11}$, or about as large as those recorded from the recent, huge, toroidal assemblies with cubic meter size of the magnetically confined fusion plasma. In contrast, the focus emission originates from one or sometimes several tiny reaction volumes, order cubic mm, and it goes on at a rate that is many orders of magnitude faster, typically an emission duration of 100 nsec compared to 1 msec. Still, the refined experiment diagnostics prove that the plasma focus fusion reaction mechanisms do rely upon some kind of dynamic but yet magnetic plasma confinement. No direct physical relationship has been found to the other mainline approach to achieve fusion energy, i.e. the purely inertial confinement by means of intense laser or particle beam compression of small fusion fuel pellets.

There are several reasons for the slight interest in plasma foci as compared to the two mainline approaches. One concerns fusion reactor aspects and involves, in addition to technological problems,

intricate deliberations about “dirty” fission-fusion hybrid concepts compared to “pure” fusion energy production. Another reason in the persistent lack of a generally acceptable physical explanation for the processes occurring in the focus plasma in spite of a large and increasing body of detailed and mostly concurring experimental observations. With reference to certain empirical but well-established scaling laws between the neutron emission and central discharge parameters three experienced focus scientists [2] recently went as far as to suggest that a fundamental but not understood physical phenomenon is probably operative during the so-called dissipative phase when the intense but still remarkably steady neutron emission occurs.

In a recent unpublished report “Mechanisms creating nuclear fusion in the dense plasma focus” we have given arguments for just such a physical phenomenon. The present work presents the basic theory given in that report but in a more accessible and general form. Crudely, the chief mechanism is a plasma ion acceleration, basically of the betatron type, driven by the magnetic variations associated with that partial discharge current disruption, incorrectly usually referred to as a $m = 0$ (“sausage”) instability, which always precedes the dissipative phase. The mechanism, paradoxically, conflicts with “well-established” plasma theory and is also contained in the generally accepted theoretical plasma fluid description. The contradiction will be traced back to a fundamental but overlooked fact: Quasi-

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neutrality in a magnetized plasma is sustained by internal forces of *non-central* character, thus invalidating a generally accepted mechanical theorem on angular momentum conservation.

Detailed comparisons with focus experimental results as well as suggestions about modifications and improvements will be deferred to a later publication. There will be several references here to the fundamental plasma theory presented in an extensive and recent review "Ideal magnetohydrodynamic (MHD) theory of magnetic fusion systems" [3]. This review, in effect, summarizes the accumulated theoretical knowledge from three decades of magnetic fusion research, and it will provide a background for our differing plasma description from which alternative approaches arise in achieving magnetic fusion. In addition, the review stresses some surprising estimates about the time scales necessary and available to achieve fusion conditions in magnetic confinements. In essence, "ideal MHD theory" is claimed to be nonvalid for magnetically confined plasmas of fusion interest. As will be discussed, there is probably better experimental than theoretical support for this view, anyhow, it provides an incentive for alternative approaches to achieve nuclear fusion by electromagnetic techniques.

II. General Plasma Fluid Equations

As described in basic texts [4] on plasma physics the starting point in derivations of plasma fluid equations is the full set of the Maxwell equations, in SI units and with standard notations

$$\text{curl } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad (1)$$

$$\epsilon_0 \text{div } \mathbf{E} = \rho_e, \quad (2)$$

$$\text{curl } \mathbf{B} = \mu_0 \mathbf{j}, \quad (3)$$

$$\text{div } \mathbf{B} = 0. \quad (4)$$

The absence of a displacement current in (3) implies, essentially, that characteristic plasma velocities are taken to be much less than the speed of light. These equations have to be coupled with a kinetic model of the plasma as described by the Boltzmann equation for each plasma species. Then, by taking appropriate moments equations for the conservation of mass, momentum and energy are obtained. E.g., the exact mass transport equations for singly ionized

ions and electrons, respectively, in a highly ionized plasma are found from a simple combination of the first and the second moments. In usual notations the result is expressed as

$$n_i m_i \frac{d\mathbf{V}_i}{dt} = e n_i (\mathbf{E} + \mathbf{V}_i \times \mathbf{B}) - \nabla \cdot \mathbf{P}_i + \mathbf{R}_i, \quad (5)$$

$$n_e m_e \frac{d\mathbf{V}_e}{dt} = -e n_e (\mathbf{E} + \mathbf{V}_e \times \mathbf{B}) - \nabla \cdot \mathbf{P}_e + \mathbf{R}_e. \quad (6)$$

\mathbf{R}_i represents the fiction force upon the ionic motion by collisions with other plasma species, and \mathbf{R}_e is to be interpreted similarly. From *linear* momentum conservation it follows

$$\mathbf{R}_i + \mathbf{R}_e = 0. \quad (7)$$

Note the non-linear character of the convective velocity derivatives, and their expansion by the Lamb transformation

$$\begin{aligned} \frac{d\mathbf{V}_i}{dt} &= \frac{\partial \mathbf{V}_i}{\partial t} + (\mathbf{V}_i \cdot \nabla) \mathbf{V}_i \\ &= \frac{\partial \mathbf{V}_i}{\partial t} + \frac{1}{2} \text{grad } V_i^2 - \text{curl } \mathbf{V}_i \times \mathbf{V}_i. \end{aligned} \quad (8)$$

If the electron inertia term in (6) is taken to be negligible the addition of (5) and (6) yields the familiar single fluid MHD equation for mass motion

$$\rho_m \frac{d\mathbf{V}}{dt} - \rho_e \mathbf{E} - \mathbf{j} \times \mathbf{B} + \nabla \cdot \mathbf{P} = 0. \quad (9)$$

For a fully ionized plasma, clearly, the mass density obeys $\rho_m = m_i n_i$. Because of the good mechanical coupling between the heavy species of a partially ionized plasma the inertia term in (9) is normally taken to refer to a total mass transport driven by the remaining terms. The use of "ion-slip" corrections to central plasma parameters is often a convenient way in justifying, or at least improving this simplification, so as to avoid a separate description of the neutral species motion [5, 8].

The excess charge density $\rho_e = e(n_i - n_e)$ is practically always taken negligible, and correctly so, however, the arguments for doing this are both fundamental and complicated [4] (see, in particular, the brilliant discussion by Chen [4] on the paradoxical "plasma approximation" $n_i = n_e$ but $\text{div } \mathbf{E} \neq 0$). In short, (5) and (6) assign widely differing characteristic time and length scales for the motion of electrons and ions. These have to be reconciled with the strong and dynamic restriction $n_i = n_e$ implied

by limits on energy, momentum as well as plasma particle dynamics. Apparent contradictions then easily arise. E.g., the normal ability of excess charge electrons to move so as to provide average charge neutralization on the tiny Debye length scale is in contrast with the much larger curvature radius in their magnetically governed motion. Further, according to first order orbit theory this drift motion will not necessarily be directed so as to neutralize the excess charge.

The pressure tensor \mathbf{P} in (9) is simply the sum of the corresponding species tensors \mathbf{P}_i and \mathbf{P}_e , and the current density \mathbf{j} is

$$\mathbf{j} = e n_i \mathbf{V}_i - e n_e \mathbf{V}_e = e n_e (\mathbf{V}_i - \mathbf{V}_e) + q_e \mathbf{V}_i. \quad (10)$$

With the discussed simplifications, i.e. $q_e \approx 0$, the ion velocity \mathbf{V}_i taken equal to the averaged mass velocity \mathbf{V} and the electron fluid inertia neglected the electron fluid (6) can be cast with the aid of (10) in the form of the familiar Generalized Ohm's law

$$\mathbf{E} + \mathbf{V} \times \mathbf{B} = \frac{1}{e n_e} \mathbf{j} \times \mathbf{B} - \frac{1}{e n_e} \nabla \cdot \mathbf{P}_e + \frac{1}{e n_e} \mathbf{R}_e. \quad (11)$$

For high density plasmas, the friction \mathbf{R}_e can safely be taken directed as the velocity difference between the ions and the electrons, i.e. the current density \mathbf{j} (10), so that

$$(1/e n_e) \mathbf{R}_e = \eta \mathbf{j}, \quad (12)$$

where the constant of proportionality η is defined as the plasma scalar resistivity. Its inverse, the scalar conductivity σ can generally be expressed as the product of the plasma charge density and a mobility μ_e , here

$$\sigma = e n_e \mu_e, \quad (13)$$

$$\mu_e = e \tau_{ei} / m_e. \quad (14)$$

Note that the electronic mobility μ_e is readily modified [5, 8] so as to include the ionic contribution to it in case the mass velocity \mathbf{V} would differ from the ionic velocity \mathbf{V}_i . τ_{ei} is usually explained as the time between momentum-randomizing collisions with ions for the average electron. Actually, it is a corresponding effect but caused by cumulative small-angle scatterings.

Provided τ_{ei} is long enough electrons can gyrate magnetically. The Hall parameter

$$\mu_e B = \omega_{ge} \tau_{ei}, \quad \omega_{ge} = e B / m_e \quad (15)$$

expresses the number of radians passed through in such gyrations between collisions. Under magnetic fusion conditions the Hall parameter is a large number. In contrast, the magnetic deflection of individual charge carriers in metals or semiconductors, i.e. the usual Hall effect, is normally small or negligible, in spite of the fact that their scalar conductivities are equal to or even exceed by orders of magnitude those of fusion plasmas. This fact illustrates a fundamental but often overlooked duality in the concept of high or nearly infinite conductivity. It can arise from a high density of charge carriers with a limited mobility but also from the reverse situation with the ratio μ_e/σ still finite. Standard concepts like plasma "frozen" to magnetic field lines or slow diffusion of magnetic flux strictly applies only to the former case which unfortunately remains fundamental for the magnetic fusion theory reviewed in [3], to be critically examined here in the following sections.

III. Simplified Plasma Fluid Equations

In order to be of practical use various simplification usually denoted as MHD approximations have to be imposed upon the basic equations (9) and (11). [3] explains clearly, in considerable detail and with references to extensive treatises the arguments leading a very simplified set of equations denoted as "ideal MHD equations". To most of these well-established reasonings and results, e.g. neglect of the excess charge q_e in (9) thereby making (2) redundant, and the retaining of only the scalar pressure parts of the pressure tensors, any objections can hardly be raised. However, in the "ideal MHD equation" form of the Generalized Ohm's law (11) *all* the three RHS terms are taken negligible. It will be shown here that this is not an acceptable approximation. In particular, it will be proved that the neglected first RHS term, the Hall term, can *never* be neglected under conditions generally acknowledged to be typical of more or less dynamic magnetic fusion containment.

In assessing the relative magnitudes of the three RHS terms of (11) there will be needed, in addition to the characteristic values E , V and B provided by the LHS terms a characteristic length L , either, following Shkarofsky et al. [4], obtained from (3) as

$$\mu_0 L j = B \quad (16)$$

or, like [3], from a quasi-static version of (9) as

$$L j B = p, \quad p \approx n_i k_B T. \quad (17)$$

Note that the origin of the estimate (16) is more general than that of (17) and, further, if (16) can be shown to imply more restrictive criteria than (17), then (17) can be used for magnitude estimates between the three retained terms in the mass transport equation (9).

The criterion for neglecting the electron pressure gradient in (11) is essentially a small electron Larmor gyration radius, $r_{Le} \ll L$

$$p_e / L n_e V B \approx (r_{Le} / L) (v_e / V), \quad v_e \approx (p_e / n_e m_e)^{1/2} \quad (18)$$

provided that the velocity ratio does not attain very large values as it certainly can do but only under quasi-equilibrium situations. Shkarofsky et al. [4] rewrites (18) as an inequality, subject to (16), involving restrictions on several parameters, among them α and γ , both to be defined and discussed later. Insertion of them in the rewritten form of (18) then readily proves it to be satisfied, but hardly with any better physical insight obtained than that shown by expression (18) above. Further, it will be proved that the electron pressure gradient term does not influence the strong plasmadynamic effects introduced by the Hall term and, hence, it will not be discussed further.

The condition for neglecting the resistivity term $R_e / e n_e$ in (11) as expressed by (12) follows from (13) and (14) in combination with the scaling (16)

$$\frac{m_e}{m_i} \frac{L}{V \tau_{ei}} \left(\frac{\lambda_i}{L} \right)^2 \ll 1 \quad (19)$$

however, the same equations but instead in combination with the scaling (17) yield a different criterion which is essentially the same as that derived in [3]

$$\frac{m_e}{m_i} \frac{L}{V \tau_{ei}} \left(\frac{r_{Li}}{L} \right)^2 \ll 1. \quad (20)$$

The length λ_i in (19) is the ion collisionless skin depth

$$\lambda_i^2 = m_i / \mu_0 e^2 n_i \quad (21)$$

also called the ion plasma wave-length by its relation to the ion plasma frequency

$$\lambda_i = c / \omega_{pi}, \quad \omega_{pi}^2 = n_i e^2 / \epsilon_0 m_i. \quad (22)$$

The electronic counterparts of (21) and (22) are found, trivially, by changing m_i for m_e . The ion Larmor radius r_{Li} in (20) is the ratio of the ion thermal speed $(p_i / n_i m_i)^{1/2}$ and the ion magnetic gyration angular frequency ω_{gi}

$$r_{Li} = (p_i / n_i m_i)^{1/2} / \omega_{gi}, \quad \omega_{gi} = e B / m_i. \quad (23)$$

The choice between (16) and (17) is determined by the squared ratio

$$(\lambda_i / r_{Li})^2 = B^2 / \mu_0 p_i \approx p_{\text{magn}} / p_i \quad (24)$$

which for any magnetic fusion concept necessarily and considerably exceeds unity. Consequently, (17) and the inequality (20) have to be discarded in favor of the more restrictive (16) and (19).

The inequality (19) is readily recast as the familiar large magnetic Reynolds number criterion necessary for neglecting resistive electric fields compared to inductive

$$R_m^{-1} = \eta / \mu_0 L V \ll 1. \quad (25)$$

The relative magnitude of the Hall term in (11) is obtained with the aid of (16) together with (13)–(15) as

$$\gamma = j B / e n_e V B = B / e n_e L \mu_0 V = \mu_e B / R_m \quad (26)$$

i.e. as the ratio of two large quantities, the Hall parameter and the magnetic Reynolds number. The ratio γ of (26) is still undetermined, however, a comparison between (25) and (26) directly proves that the neglect of the Hall term is by the factor $\mu_e B$ a far more restrictive simplification than the neglect of resistivity.

It is surprising that the importance of the Hall term seems fully recognized in theories of MHD power generation and MPD plasma acceleration. Typically, partitioning of electrodes is a standard technique so as to avoid excessive resistive losses which otherwise would follow from Hall effect-induced distortions in current and potential distributions. In magnetic fusion context, however, “ideal MHD theory” is often taken to be “augmented” by solely the inclusion of resistive effects.

In evaluating the ratio γ (26), it is first noted that the proven less restrictivity of the scaling (17) as compared to (16) indicates that the inertia term in (9) has to take part in balancing the $j \times B$ force. Accordingly, it will be assumed

$$\left| q_m \frac{dV}{dt} \right| \approx q_m V^2 / L \approx B^2 / \mu_0 L, \quad \text{i.e. } \alpha = B^2 / q_m \mu_0 V^2 \approx 1. \quad (27)$$

As discussed by Shkarofsky et al. [4] this implies that attention is rather given to strong and arbitrary mass motions than those linearly perturbative motions typical of wave propagation.

The full criterion for neglecting the Hall term in the Generalized Ohm's law (11), subject to $\alpha \approx 1$ has been derived by Braginskii [6]. His derivation need not be repeated here as the identical result will be derived in different ways together with physical interpretations in the sequel. Braginskii's criterion reads

$$\gamma^2/\alpha = \lambda_i^2/L^2 \ll 1 \quad (28)$$

i.e. the Hall term is negligible if the ion collisionless skin depth is much smaller than the plasma characteristic length.

To see how well this criterion is satisfied we may accept an estimate presented as follows in [3]: "Past experiments and extrapolations to future fusion reactors indicate that the densities ... of fusion plasmas lie in the range

$$10^{12} < n < 10^{16} \text{ cm}^{-3}. \quad (29)$$

(End of citation.) Of this range (we note that) the lower part, $10^{12} - 10^{14} \text{ cm}^{-3}$, mainly refers to confinement of the toroidal and mirror geometries. For a fully ionized and pure deuterium plasma the corresponding λ_i -values will range between 0.3 m and 3 cm, but a few atomic percent impurities, thermonuclear fusion fuel heavier than deuterium or incomplete ionization can easily make the λ_i -values considerably larger. Present toroidal and mirror research assemblies contain plasmas with pertinent scales that simply do not satisfy the inequality (28). Equality, instead, is a more reasonable approximation! The higher part of (29), i.e. the range $10^{14} - 10^{16} \text{ cm}^{-3}$, most likely refers to more compact assemblies like the theta-pinch, the spheromak and the high-beta stellarator. Their characteristic variation lengths for properties of the confined plasma hardly exceed the corresponding λ_i -values, 3 cm to 3 mm. Again, equality in (28) would be more reasonable, and there might well arise a suspicion that the ubiquitous ion collisionless skin depth λ_i signifies some basic plasma properties overlooked by "ideal MHD theory". Indeed, that will be shown in the following.

IV. Plasma Rotation

In [7] it was first shown that the two basic MHD fluid equations (9) and (11) can be combined, with

the aid of the Maxwell equations and the expansion in [8] so as to yield, without approximations, an expression that relates magnetic flux variations with plasma mass rotation

$$\begin{aligned} \frac{d}{dt} \oint \mathbf{B} \cdot d\mathbf{S} = & - \frac{d}{dt} \oint \frac{q_m}{e n_e} \mathbf{V} \cdot d\mathbf{s} \\ & + \oint \left[\frac{q_e}{e n_e} \mathbf{E} - \frac{1}{e n_e} \nabla \cdot (\mathbf{P} - \mathbf{P}_e) - \frac{\mathbf{j}}{\sigma} \right] \cdot d\mathbf{s}. \end{aligned} \quad (30)$$

The integrals refer to a closed loop, length element $d\mathbf{s}$, defining the boundary of a simple closed surface, surface element $d\mathbf{S}$. The loop is attached to the plasma mass frame, i.e. the loop is "frozen" into the plasma so that it follows its motion. The derivation is not too difficult by starting with the identity

$$\begin{aligned} \frac{d}{dt} \oint \mathbf{a} \cdot d\mathbf{S} = & \oint \frac{\partial \mathbf{a}}{\partial t} \cdot d\mathbf{S} \\ & + \oint \text{div } \mathbf{a} \mathbf{V} \cdot d\mathbf{S} - \oint \mathbf{V} \times \mathbf{a} \cdot d\mathbf{s} \end{aligned} \quad (31)$$

applied to the vector field

$$\mathbf{a} = \mathbf{B} + \frac{q_m}{e n_e} \text{curl } \mathbf{V}. \quad (32)$$

In the gas dynamic limit $n_i = n_e \rightarrow 0$ (30) reduces to the Kelvin circulation theorem expressing the approximate conservation of mass vorticity

$$\frac{d}{dt} \oint \mathbf{V} \cdot d\mathbf{s} = - \oint \frac{1}{q_m} \nabla \cdot \mathbf{P} \cdot d\mathbf{s} \approx 0 \quad (33)$$

however the opposite limit $n_i = n_e \rightarrow \infty$ does not lead to the classical Alfvén flux conservation theorem

$$\frac{d}{dt} \oint \mathbf{B} \cdot d\mathbf{S} \approx 0. \quad (34)$$

To obtain this result the drastic "zero ion Larmor radius" condition $m_i = 0$ is usually argued additionally. A better way would be to have the factor $1/e n_e$ in the Hall term of (11) replaced by the discussed ratio μ_e/σ or its "ion-slip" modified version $\mu_e(1-x)/\sigma(1+x)$ where x is the ratio of ionic and electronic mobilities [8]. Then, classical flux conservation (34), is obtained for the mentioned metal or semiconductor case, i.e. $\sigma \rightarrow \infty$ with μ_e still being finite.

The magnitude estimates in the preceding section indicate that the last integral in (30) is small under plasma fusion conditions. In order, the integrand terms mean that a rotational electric field acting on a plasma with excess charge will, trivially, make

such a plasma rotate, and that ion viscosity like ion-electron friction will retard mass rotation. The scalar pressure part of the tensor difference term is essentially of gradient character, i.e. non-contributing. However, under rather special circumstances

$$\text{grad } n_i \times \text{grad } T_i \neq 0 \quad (35)$$

this so-called baroclinic vector may generate magnetic flux or drive rotation. Stamper et al. [9] rediscovered the corresponding field-generating ability of the plasma electron pressure and applied it to laser plasmas. With reference to less extreme plasma conditions Cowling [10] assigns importance to the same effect only in generating the small seed field necessary for various but different dynamo mechanisms.

With the RHS line integral in (30) negligible this equation merely expresses conservation of canonical, i.e. matter-plus-field, angular momentum for the coupled plasma heavy constituents. Thus, plasma mass rotation can give rise to magnetic flux. In the very first attempt to create spheromak plasma configurations, then modestly called plasma rings, a remarkable poloidal flux generation was observed [11]. This flux generation mechanism was later found to be uniquely associated with a toroidal plasma rotation which, as described by (30), transformed into poloidal magnetic flux [12]. Spontaneous magnetic field generation has also and often been observed in laser plasmas [9]. Hasegawa et al. [13] recently derived a simplified differential form of (30) from the ionic species equation of motion alone, (5) here, and applied it to a description of laser plasmas as magnetized vortex structures with the ion collisionless skin depth λ_i as the derived characteristic dimension. The same topic but with more detailed theoretical justification and references to experimental observations was treated in [14].

Consider the very simplest case of a magnetic flux $\Phi_z(r)$ with uniform field strength B_z which changes in time so as to cause an azimuthal rotation $V_\phi(r)$ according to the simplified (30). Such a *momentum* transfer from field to matter can be indicated as

$$\Phi_z(r) = \pi r^2 B_z \rightarrow \frac{Q_m}{e n_e} 2\pi r V_\phi(r), \quad Q_m = n_i m_i \quad (36)$$

and squared, this can be expressed as the *energy* transfer

$$\frac{B_z^2}{2\mu_0} \rightarrow 4 \frac{\lambda_i^2}{r^2} \cdot \frac{1}{2} Q_m V_\phi^2 \quad (37)$$

which obviously requires $r \approx \lambda_i$ in order to be energetically possible. Physically, (36) and (37) just describe a betatron acceleration mechanism but acting on plasma ions. Even the factor 4 in (37) is nothing but the square of that very figure two appearing in the classical betatron 2:1 field rule.

We denote the effect of this betatron type plasma ion acceleration as magnetically induced plasma rotation. Its applicability range is very wide, and we have used it to explain strongly pronounced phenomena in magnetized plasma contexts as diversified as MPD accelerators, MHD power generators, laser sparks and coaxial plasma guns. Probably, its best proof of validity is given in an abundance of well-established experimental observations on theta pinches. The theory [15] not only explains their more evident features like their fast rotation, the early rotation onset coincident with the externally applied field penetration, the preferential energy transfer to the ions and the field-plasma interaction range always observed to agree with the ion collisionless skin depth λ_i [16]. Slight extensions of the theory [15] yield explanations to more intricate properties previously not understood like the peculiar influence from the sign of a bias field, separations of gradients in magnetic field and plasma density and even an observation of rotation reversal.

V. The Electron Fluid Equation

The basic (30) describes plasma ionic motion and, in essence, it is just an integral form of the equation for ion mass transport (5), but with the neutral species attached to the ions. Note that there is no neglect of electron inertia in (30) in contrast to (9) and (11). The electron fluid equation (6), is readily rewritten with the aid of (1), the identity (31) and an identity for V_e similar to (8) as

$$\frac{d}{dt} \oint \mathbf{B} \cdot d\mathbf{S} = \frac{d}{dt} \oint \frac{m_e}{e} \mathbf{V}_e \cdot d\mathbf{s} + \oint \frac{1}{e n_e} \nabla \cdot \mathbf{P}_e \cdot d\mathbf{s}. \quad (38)$$

The integrals here refer to a loop attached to the moving electron fluid. With $m_e \approx 0$ (38) expresses the Lighthill [17] theorem of approximate magnetic flux conservation in electron fluid frame, not mass frame, and expansions of the last term yield the mechanism for spontaneous magnetic generation proposed by Cowling [10] and Stamper et al. [9].

Observe that the validity of (38), in contrast to (30), is dependent upon the quasi-neutrality condi-

tion $n_e = n_i$ being satisfied. Further, and paradoxically, quasi-neutrality can by no means be taken to imply negligible space charge field effects. This is discussed in text-books [4] and also in [3], see in particular the clear discussion with emphasis on this point by Chen [4]. In short, neutralization by electrons is easily achieved on the ion fluid characteristic length and time scales, λ_i and ω_{pi}^{-1} , respectively. The opposite case, neutralization of the electron fluid by ionic motion, can neither occur on the characteristic electronic length scale or "sharp boundary" thickness $\lambda_e = c/\omega_{pe}$, nor on the short electronic time scale ω_{pe}^{-1} . The result is that quasi-neutrality compels the electronic motion loop considered in (38) to remain coincident with the ionic one described by (30), however, a central question remains: How do the charged species move in their rotational motions along these coincident closed loops? Quasineutrality then enters again, but in a far more subtle way, to be discussed in section VI.

With a proper handling of space-charge field effects the equation pair (5) and (6) or, equivalently (9) and (11) or (30) and (38) must of course describe a plasma correctly. As an example, we shall derive theoretically the experimentally observed "diffuse" plasma boundary thickness λ_i of the usual fast theta pinch [16]. The incorrect use of an electronic species equation of motion alone, like (6) or (11) or (38), leads to the "skin current" or "sharp boundary" value λ_e . Various instabilities or "anomalous" effects are usually then invoked in order to bring agreement [16], [18] with the "diffuse" value λ_i .

Consider the simple geometry described in the previous derivation of λ_i (37). With the discussed simplifications (6) or (30) gives

$$m_i \frac{d}{dt} (V_{i\phi} r) = -e \frac{d}{dt} \int_0^r B(\xi) \xi d\xi. \quad (39)$$

The Maxwell equation (3) is expressed as

$$\text{curl } \mathbf{B} = \mu_0 (\mathbf{j} + \text{curl } \mathbf{M}) \quad (40)$$

which means that we distinguish between charge transport by free charges, denoted by \mathbf{j} , and that by the magnetically bound or "static", also called "diamagnetic" particle motion, where the magnetization \mathbf{M} is given by the density of the magnetic moments v_e of gyrating electrons

$$\mathbf{M} = -p_{e\perp} \mathbf{B}/B^2, \quad p_{e\perp} = \frac{1}{2} n_e m_e v_{e\perp}^2 = n_e v_e B. \quad (41)$$

This partition of free, \mathbf{j} , and bound, $\text{curl } \mathbf{M}$, current density, basic in classical theory of magnetized media, is of course generally valid, however, the advantages in plasma contexts are not always realized.

For the assumed cylindrical geometry the Maxwell equation (40) becomes

$$-\frac{1}{\mu_0} \frac{\partial B}{\partial r} = j_\phi + \frac{\partial}{\partial r} \left(\frac{p_{e\perp}}{B} \right), \quad (42)$$

where the free particle current is the sum of the azimuthal ion motion and the corresponding component of the current by electronic motion. If particle orbit theory is taken to describe the latter only the electric field drift and the magnetic field gradient drift enter for these drift motions

$$j_{e\phi} = e n_e \frac{E_r}{B} + \frac{p_{e\perp}}{B^2} \frac{\partial B}{\partial r} \quad (43)$$

and the free particle current becomes

$$j_\phi = e n_i V_{i\phi} + j_{e\phi}. \quad (44)$$

(42), (43) and (44) give the total current

$$-\frac{1}{\mu_0} \frac{\partial B}{\partial r} = e n_i V_{i\phi} + e n_e \frac{E_r}{B} + \frac{1}{B} \frac{\partial p_{e\perp}}{\partial r}. \quad (45)$$

Of course, the electronic part of this current might as well have been obtained directly from the electron fluid (6) with electron inertia, viscosity and resistivity neglected. A non-trivial observation, however, is that the field gradient part $-\frac{p_{e\perp}}{B^2} \frac{\partial B}{\partial r}$ of the

"static" magnetization current in (42) is identically cancelled by the "free" magnetic gradient drift current in (43). This paradoxical cancellation was analysed by Tonks [19] by a most ingenious reasoning in an early and, apparently, forgotten work. He explains the cancellation as the effect of the cyclic length changes in the Larmor orbit radii of the magnetically gyrating electrons in an inhomogeneous field. It does not seem possible reproduce this reasoning and the result as some kind of guiding center shifting or motion in the usual particle drift motion formulation. As similar cancellation can be proved and is also readily seen to occur between the electron pressure part $\frac{n_e}{B} \frac{\partial}{\partial r} \left(\frac{m_e}{2} v_{e\perp}^2 \right)$ of the "static" magnetization current in (42) and the "free" electron drift motion in (43). Consider the expanded

Eq. (45)

$$-\frac{1}{\mu_0} \frac{\partial B}{\partial r} = e n_i V_{i\varphi} + \frac{n_e}{B} \left[e E_r + \frac{\partial}{\partial r} \left(\frac{m_e}{2} v_{e\perp}^2 \right) \right] + v_e \frac{\partial n_e}{\partial r}. \quad (46)$$

The change of electron energy gained from the electric field, $-e E_r$, is balanced by the corresponding change in electron kinetic energy, and the bracket terms in (46) cancel. Elimination of $V_{i\varphi}$ between (39) and (46) then yields

$$\frac{d}{dt} \left[\lambda_i^2 \left(\frac{\partial^2 \Phi}{\partial r^2} - \frac{1}{r} \frac{\partial \Phi}{\partial r} + 2 \pi \mu_0 v_e r \frac{\partial n_e}{\partial r} \right) - \Phi \right] = 0, \quad \Phi(r) = 2 \pi \int_0^r B(\xi) \xi d\xi. \quad (47)$$

In [15] we have discussed and presented solutions to this differential equation, and also proved that the last term of (46), i.e. the truly “static” part of the diamagnetic current is, compared to the flux terms, insignificant in fusion plasma contexts. The time derivative in (47) then operates on a quantity proportional to the ionic canonical angular momentum, and (47) also proves this to be conserved. More important, the bracket differential equation expression for this conserved canonical angular momentum directly shows, without need for the detailed solution [15], that the ion collisionless skin depth $\lambda_i(r)$ is the characteristic length parameter for variations in the magnetic flux $\Phi(r)$.

VI. Electromagnetic Torque Distribution on a Plasma

As mentioned, if the theoretically derived value $\lambda_i \gg \lambda_e$ is taken as a general plasma-field interaction distance conflicts arise with accepted plasma theory, often concerned with “sharp-boundary” models and processes for widening as assumed λ_e into the observed λ_i [16, 18]. There is, but seldom realized or mentioned, a severe and incorrect restriction behind these works. To explain it see (39) that describes the driving effect on ions from the inductive electric field associated with a magnetic flux variation. The combined effect on ions and electrons by the very same inductive field was derived from both particle orbit and fluid theory, and is given by (45) and (46). The current is largely ionic which means that the mechanical angular momentum transferred to the

plasma by the inductive electric field will be imparted almost exclusively upon the positive ions. This occurs in spite of the fact that there is almost exactly as much negative charge from electrons everywhere in the plasma. If one takes this charge neutrality as the argument for the electronic mass motion to balance mechanically the ionic angular momentum then, clearly, only strong thin and flux-preserving electronic skin currents can exist, in accordance with the Lighthill “frozen-field”, see (38).

Essentially the same effect by charge-neutrality is expressed in a formal disproof of the possibility for the electromagnetic torque to bring rotation of any significant magnitude to a plasma column [20]. Actually, this result is the origin of the persistent controversy about the origin of rotation in magnetically confined plasmas [21]. The disproof presumes purely single-fluid properties of the plasma and it concerns a theta pinch plasma volume which is azimuthally symmetric, globally but not locally charge-neutral and which has no in- or out-flows of currents through the bounding surfaces. The total electromagnetic torque can be expressed by the Maxwell stress tensor

$$\mathbf{T}_e = \left(\frac{1}{2} \varepsilon_0 E^2 + \frac{1}{2 \mu_0} B^2 \right) \mathbf{1} - \varepsilon_0 \mathbf{E} \mathbf{E} - \frac{1}{\mu_0} \mathbf{B} \mathbf{B} \quad (48)$$

and the electromagnetic momentum

$$\mathbf{G}_e = \varepsilon_0 \mathbf{E} \times \mathbf{B} \quad (49)$$

as the volume integral taken over the column

$$\int \mathbf{r} \times (Q_e \mathbf{E} + \mathbf{j} \times \mathbf{B}) d\tau = - \int \mathbf{r} \times \left(\nabla \cdot \mathbf{T}_e + \frac{\partial \mathbf{G}_e}{\partial t} \right) d\tau. \quad (50)$$

As it can be shown [20] that the RHS tensor divergence terms vanish upon integration the conclusion is that only the exceedingly minute electromagnetic momentum can drive rotation.

Even accepting the single fluid plasma approximation the disproof is fundamentally incorrect because it makes use of a non-valid continuum treatment of a system possessing internal angular momenta, here mainly gyrating electrons. E.g., from the disproof it would follow, as one may also expect intuitively, that an increase of global plasma rotation will be counteracted, in the absence of any external net torque or any angular momentum transport across the boundaries, by an increase of op-

positely directed internal angular momenta. In the discussion of stress tensor symmetry properties [22] studies this situation by means of admittedly advanced tensor formalism, but the essential outcome of the analysis is simple: Intuition goes wrong!

Before analysing the electromagnetic torque distribution it is pointed out that a plasma is a particle system, not a fluid, and the absence of a net torque on a particle system like a quasi-neutral plasma does *not* imply the conservation of its mechanical angular momentum. Conservation would require, in addition, exclusively central force particle-particle interactions. The electromagnetic forces which *sustain the quasineutrality* among magnetically gyrating plasma particles are not of the central force kind. [23] has given an eloquent and devastating criticism of practically all text-books for neglecting or not paying proper attention to this central force interaction restriction in discussions on torques and angular momenta.

There is no way so as to integrate the Boltzmann equation over an angular velocity space. Further, the concept of a convective derivative applied to a moving plasma volume is of course devoid of meaning if the motion of its electronic part differs from that of its ionic. The proper treatment is explained by Penfield and Haus [22], and their stress tensor symmetry analysis will be extended here to a system with two kinds of charged particles, ions and electrons. The proper momentum variable must be that per particle \mathbf{G}/n

$$\mathbf{G} = \mathbf{G}_k + \mathbf{G}_e, \quad \mathbf{G}_k = \varrho_m \mathbf{V}, \quad \mathbf{G}_e = \varepsilon_0 \mathbf{E} \times \mathbf{B},$$

$$n = n_i + n_e, \quad n_i = n_e. \quad (51)$$

The torque acting on a unit volume of plasma is given formally by

$$n \frac{d}{dt} (\boldsymbol{\sigma} + \mathbf{r} \times \mathbf{G}/n) = \nabla \cdot (\mathbf{T}^0 \times \mathbf{r}), \quad (52)$$

where $\mathbf{T}^0 = \mathbf{T} - \mathbf{V}\mathbf{G}$ is the total stress tensor in the fluid frame. The laboratory frame stress tensor \mathbf{T} is the sum of \mathbf{T}_e , the pressure tensor \mathbf{P} and the kinetic tensor $\mathbf{T}_k = \varrho_m \mathbf{V}\mathbf{V}$. $\boldsymbol{\sigma}$ is that individual plasma particle angular momentum, denoted by Penfield and Haus as intrinsic, which does not contribute upon usual velocity space integration to the average momentum flow \mathbf{G}_k . The purely electromagnetic momentum \mathbf{G}_e is entirely negligible in contexts of magnetic fusion plasma dynamics. Then

$$\mathbf{G} = n_i m_i \mathbf{V}_i + n_e m_e \mathbf{V}_e \quad (53)$$

and the RHS of (52) includes everything that drives rotation of a plasma fluid element. The important step now is *to make use of the quasi-neutrality* $n_i = n_e$ so as to split up the LHS of (52)

$$n_e \frac{d}{dt} (\boldsymbol{\sigma}_e + \mathbf{r} \times m_e \mathbf{V}_e) + n_i \frac{d}{dt} (\boldsymbol{\sigma}_i + \mathbf{r} \times m_i \mathbf{V}_i) = \nabla \cdot (\mathbf{T}^0 \times \mathbf{r}). \quad (54)$$

Note that the first convective derivative refers to the trajectory of the individual electron in the fluid element, the second to the corresponding ion. For any magnetically gyrating and drifting electron, peculiar velocity \mathbf{V}'_e , the first derivative in (54) operates on essentially a constant of motion, actually proportional to the individual magnetic moment \mathbf{v}_e

$$\frac{d}{dt} (\mathbf{r} \times m_e \mathbf{V}'_e) \sim \frac{d}{dt} (\mathbf{v}_e) \approx 0, \quad (55)$$

however, the second derivative in (54) never vanishes because ionic inertial effects are *never* negligible under conditions of global plasma rotation. Simply, (54) expresses a *preferential* mechanical angular momentum transfer from the inductive electric field to the plasma heavy species, exactly as contained in (5), or expressed by (30), or (39) and, as repeatedly pointed out, observed experimentally.

VII. Alternative Approaches to Magnetic Fusion

[3] strongly emphasizes a surprising limitation in the considered "Ideal MHD theory of magnetic fusion systems", however it is neither the serious neglect of the Hall term in the generalized Ohm's law (11), nor the other simplifications in the mass and charge transport equations. The deficiency concerns instead limits in energy transfer in fusion plasmas, specifically the small value of the characteristic plasma "life time" L/v_i where $v_i \approx (p_i/n_i m_i)^{1/2}$. This time, about the ion thermal transit time across the characteristic plasma dimension L , is typical for growth times of large-scale instabilities. [3] compares this time with the wellknown Spitzer electron-ion equilibration time τ_{ei} which necessarily must be considerably shorter than L/v_i if an external electron-heating arrangement will have sufficient time to heat the plasma up to thermonuclear conditions. Because of the shorter ion-ion equilibration time τ_{ii} the situation will improve, but not sufficiently, with some heating acting directly on the ions.

It has been shown by use of the Chandrasekhar – Fermi – Schmidt virial theorem for plasmas, but in contexts of refuting plasmoid explanations of ball lightnings [24], that the characteristic magnetic confinement “life-time” L/r_i is nothing but the short disintegration time for a plasma entity in the absence of any confinement! The same virial theorem, far more general than “ideal MHD”, can readily be extended so as to predict longer times L/r_i for inertial fusion schemes of the implosion type and, of course, arbitrarily long times for stable magnetic confinement [25]. Still, experimental results vindicate [3] in the estimate L/r_i which can even be too optimistic. The feared disruptive instabilities ruin magnetic plasma confinement at rates faster than indicated by the mathematically diffusive character inherent in “ideal MHD theory” but not in MHD theory with the Hall term retained [17].

At present there is strong interest in new magnetic confinement geometries, in particular the spheromak with a theoretically expected improved stability [26]. As described in [12] such toroidal plasmas with a poloidal confining field can be created if there is an initial toroidal plasma rotation, in accordance with (30). Additional and strong arguments have been given in favor of these compact tori [26], however, the necessarily drastic increase over L/r_i in their stable confinement time remains to be proved experimentally.

The dense plasma focus operates so as to defy almost any accepted provision on magnetic fusion. E.g., its operation is affected by plasma impurities but not necessarily in a negative way. Also, a disruptive instability, actually a positive feed-back current choking mechanism relying on effects contained in (30), is a prerequisite for the ensuing dissipative or diffuse phase with its copious neutron production. About this focus scientists have agreed on the following statement [27]: “Converging experimental evidences point to the following model: most neutrons are due the interaction with low density (10^{17} – 10^{18}) plasma structures of medium energy (≈ 100 keV) ions confined for a long period of time in a self-sustained magnetic configuration. Further experimental work, such as high time and space resolution of the neutron source, is required for the final assessment of the model.”

The expressed need for an improved understanding by experimental means is well-founded. (Most recently, the initial ion acceleration mechanism was

identified as originating from electric fields and acting on deuteron ions and impurities in common processes [28].) However, the statement can also be interpreted as the description of an intensely fusion-reacting plasma, reasonably stable, magnetically confined and relying upon a built-in ion acceleration and heating mechanism. Thus, the focus plasma provides more than intense neutron bursts, it also indicates alternative approaches to magnetic fusion. We intend to recur to this in a forth-coming publication.

VIII. Summary and Conclusions

After some introductory views on mainline vs. alternative approaches to magnetic fusion the usual plasma fluid equations were derived and given interpretations similar to that in good text-books [4]. In addition, a certain duality in the concept of high conductivity was pointed out. It was proved in the sequel that the high charge carrier density alternative, typical for metals, yields the essential basis for the classical and, for fusion research plasmas, mainly theoretical concept of magnetic flux “frozen-into-mass” whereas the high charge carrier mobility alternative, typical for fusion plasmas, leads after somewhat intricate reasonings based on charge-neutrality to an experimentally often observed effect that we denote as magnetically induced plasma rotation.

The discussion of simplifications in the plasma fluid equations essentially followed section 9-3 “MHD validity” in the book by Shkarofsky et al. [4]. A different result was pointed out with respect to conclusions concerning the validity of a set of “Ideal MHD Equations” as claimed in a recent extensive review “Ideal MHD Theory of Magnetic Fusion Systems” by Freidberg [3]. It concerned the importance of the Hall effect term. This was shown here to be expressed quantitatively by the relative magnitude of the ion collisionless skin depth, also called the ion plasma wavelength. This length was proved both from theoretical arguments and by data from relevant magnetic fusion experiments to be of utmost importance in magnetic fusion contexts. The same length was later shown to be a key scale parameter for conversions between magnetic and kinetic plasma energies, and it was also derived theoretically as the experimentally observed “diffuse” magnetic shielding distance in contrast to the

commonly assumed "sharp" boundary of a magnetically confined plasma. In this latter derivation we made use of a seemingly overlooked but most ingenious reasoning of Tonks [19].

Although general, the derivation of magnetically induced rotation in Sect. IV stressed the ion fluid properties the plasma, apparently in conflict with the electron fluid characteristics treated in the next section. Essentials of the resolution reproduced arguments and explanations given by Chen about the paradoxical "plasma approximation", see Chapt. 3, "Plasmas as Fluids", in his book [4].

A mathematically perfect analysis of the electromagnetic torque distribution on a plasma will probably require a strict particle-particle interaction formulation. Even by numerical computer techniques this seems out of reach, however, a usually overlooked mechanical theorem, obtainable only from such a formulation and vigorously stressed by Truesdell [23] was introductorily pointed out in the challenge of a widely accepted proof about the im-

possibility of bringing a certain plasma configuration into strong rotation by means of electromagnetic fields. The correct analysis variable and technique as explained and used by Penfield and Haus in Sect. 4.7 "Symmetry of the Stress Tensors" in their book [22], was adopted. Plasma quasi-neutrality was shown to be the key plasma property in proving the preferential action of the electromagnetic torque on the plasma ionic species.

In the final section it was pointed out that a well-founded restriction, strongly stressed by Freidberg [3] and supported both by accepted instability theories and, more so, by experiments, strictly means that conventional magnetic fusion plasma confinement equals absence of plasma confinement. The new compact tori fusion schemes were mentioned as, perhaps, one way out of this distressing situation, however, some of the remarkable features of the dense focus plasma were also pointed out to indicate that, certainly, there still exist unexplored ways in achieving magnetic fusion.

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